Unit-III Distribution System Analysis

Voltage drop and power loss calculations- 3Ø balanced primary lines

7-1 THREE-PHASE BALANCED PRIMARY LINES

utility company strives to achieve a well-balanced distribution system in order to improve system voltage regulation by means of equally loading each phase. Figure 7-1 shows a primary system with either a three-phase three-wire or a three-phase four-wire main. The laterals can be either (1) three-phase three-wire, (2) three-phase four-wire, (3) single-phase with line-to-line voltage, ungrounded, (4) single-phase with line-to-neutral voltage, grounded, or (5) two-phase plus neutral, open-wye.

7-2 NON-THREE-PHASE PRIMARY LINES

Usually there are many laterals on a primary feeder which are not necessarily in three-phase, e.g., single-phase which causes the voltage drop and power loss due to load current not only in the phase conductor but also in the return path.

Derivation of voltage drop and power loss in lines-Manual methods of solution for radial networks

7-2-1 Single-Phase Two-Wire Laterals with Ungrounded Neutral

Assume that an overloaded single-phase lateral is to be changed to an equivalent three-phase three-wire and balanced lateral, holding the load constant. Since the power input to the lateral is the same as before,

$$S_{1\phi} = S_{3\phi} \tag{7-1}$$

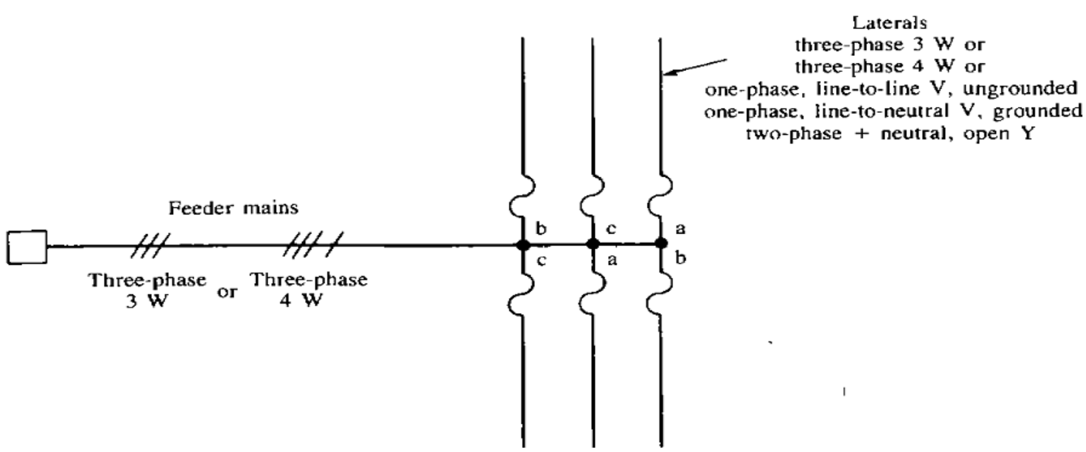

Figure 7-1

where the subscripts 1ϕ and 3ϕ refer to the single-phase and three-phase circuits, respectively. Equation (7-1) can be rewritten as

$$(\sqrt{3} \times V_s)I_{1\phi} = 3V_s I_{3\phi} \tag{7-2}$$

where V_s is the line-to-neutral voltage. Therefore, from Eq. (7-2),

$$I_{1\phi} = \sqrt{3} \times I_{3\phi} \tag{7-3}$$

which means that the current in the single-phase lateral is 1.73 times larger than the one in the equivalent three-phase lateral.

The voltage drop in the three-phase lateral can be expressed as

$$\mathrm{VD}_{3\phi} = I_{3\phi}(R\cos\theta + X\sin\theta) \qquad \mathrm{V} \tag{7-4}$$

and in the single-phase lateral as

$$\mathrm{VD}_{1\phi} = I_{1\phi}(K_R R\cos\theta + K_X X\sin\theta) \tag{7-5}$$

where K_R and K_X are conversion constants of R and X and are used to convert them from their three-phase values to the equivalent single-phase values.

$K_R = 2.0$

$K_X = 2.0$ when underground cable is used

$K_X \cong 2.0$ when overhead line is used, with approximately a $\pm 10\%$ accuracy

Therefore Eq. (7-5) can be rewritten as

$$\mathrm{VD}_{1\phi} = I_{1\phi}(2R\cos\theta + 2X\sin\theta) \qquad \mathrm{V} \tag{7-6}$$

or substituting Eq. (7-3) into Eq. (7-6),

$$\mathrm{VD}_{1\phi} = 2\sqrt{3} \times I_{3\phi}(R\cos\theta + X\sin\theta) \qquad \mathrm{V} \tag{7-7}$$

By dividing Eq. (7-7) by Eq. (7-4) side by side,

$$\frac{\mathrm{VD}_{1\phi}}{\mathrm{VD}_{3\phi}} = 2\sqrt{3} \tag{7-8}$$

which means that the voltage drop in the single-phase ungrounded lateral is approximately 3.46 times larger than the one in the equivalent three-phase lateral. Since base voltages for the single-phase and three-phase laterals are

$$V_{B(1\phi)} = \sqrt{3} \times V_{s,\,L\text{-}N} \qquad \mathrm{V} \tag{7-9}$$

and

$$V_{B(3\phi)} = V_{s,\,L\text{-}N} \qquad \mathrm{V} \tag{7-10}$$

Eq. (7-8) can be expressed in per units as

$$\frac{\mathrm{VD}_{\mathrm{pu},\,1\phi}}{\mathrm{VD}_{\mathrm{pu},\,3\phi}} = 2.0 \tag{7-11}$$

which means that the per unit voltage drop in the single-phase ungrounded lateral is two times larger than the one in the equivalent three-phase lateral. For example, if the per unit voltage drop in the single-phase lateral is 0.10, it would be 0.05 in the equivalent three-phase lateral.

The power losses due to the load currents in the conductors of the single-phase lateral and the equivalent three-phase lateral are

$$P_{L, S1\phi} = 2 \times I_{1\phi}^2 R \qquad \text{W} \tag{7-12}$$

and

$$P_{LS, 3\phi} = 3 \times I_{3\phi}^2 R \qquad \text{W} \tag{7-13}$$

respectively. Substituting Eq. (7-3) into Eq. (7-12),

$$P_{LS, 1\phi} = 2(\sqrt{3} \times I_{3\phi})^2 R \tag{7-14}$$

and dividing the resultant Eq. (7-14) by Eq. (7-13) side by side,

$$\frac{P_{LS, 1\phi}}{P_{LS, 3\phi}} = 2.0 \tag{7-15}$$

which means that the power loss due the load currents in the conductors of the single-phase lateral is two times larger than the one in the equivalent three-phase lateral.

Therefore, one can conclude that by changing a single-phase lateral to an equivalent three-phase lateral both the per unit voltage drop and the power loss due to copper losses in the primary line are approximately halved.

7-2-2 Single-Phase Two-Wire Unigrounded Laterals

In general, this system is presently not used due to the following disadvantages. There is no earth current in this system. It can be compared to a three-phase four-wire balanced lateral in the following manner. Since the power input to the lateral is the same as before,

$$S_{1\phi} = S_{3\phi} \tag{7-16}$$

or

$$V_s \times I_{1\phi} = 3 \times V_s \times I_{3\phi} \tag{7-17}$$

from which

$$I_{1\phi} = 3 \times I_{3\phi} \tag{7-18}$$

The voltage drop in the three-phase lateral can be expressed as

$$\text{VD}_{3\phi} = I_{3\phi}(R \cos\theta + X \sin\theta) \qquad \text{V} \tag{7-19}$$

and in the single-phase lateral as

$$\text{VD}_{1\phi} = I_{1\phi}(K_R R \cos\theta + K_X X \sin\theta) \qquad \text{V} \tag{7-20}$$

where $K_R = 2.0$ when a full-capacity neutral is used i.e., if the wire size used for neutral conductor is the same as the size of the phase wire.

$K_R > 2.0$ when a reduced-capacity neutral is used

$K_X \cong 2.0$ when overhead line is used

Therefore, if $K_R = 2.0$ and $K_X = 2.0$, Eq. (7-20) can be rewritten as

$$\text{VD}_{1\phi} = I_{1\phi}(2R \cos\theta + 2X \sin\theta) \qquad \text{V} \tag{7-21}$$

or substituting Eq. (7-18) into Eq. (7-21),

$$\text{VD}_{1\phi} = 6 \times I_{3\phi}(R \cos\theta + X \sin\theta) \qquad \text{V} \tag{7-22}$$

Dividing Eq. (7-22) by Eq. (7-19) side by side,

$$\frac{\mathrm{VD}_{1\phi}}{\mathrm{VD}_{3\phi}} = 6.0 \tag{7-23}$$

which means that the voltage drop in the single-phase two-wire unigrounded lateral with full-capacity neutral is six times larger than the one in the equivalent three-phase four-wire balanced lateral.

The power losses due to the load currents in the conductors of the single-phase two-wire unigrounded lateral with full-capacity neutral and the equivalent three-phase four-wire balanced lateral are

$$P_{\mathrm{LS},\,1\phi} = I_{1\phi}^2(2R) \qquad \mathrm{W} \tag{7-24}$$

and

$$P_{\mathrm{LS},\,3\phi} = 3 \times I_{3\phi}^2 R \qquad \mathrm{W} \tag{7-25}$$

respectively. Substituting Eq. (7-18) into Eq. (7-24),

$$P_{\mathrm{LS},\,1\phi} = (3 \times I_{3\phi})^2(2R) \qquad \mathrm{W} \tag{7-26}$$

and dividing Eq. (7-26) by Eq. (7-25) side by side,

$$\frac{P_{\mathrm{LS},\,1\phi}}{P_{\mathrm{LS},\,3\phi}} = 6.0 \tag{7-27}$$

Therefore, the power loss due to load currents in the conductors of the single-phase two-wire unigrounded lateral with full-capacity neutral is six times larger than the one in the equivalent three-phase four-wire lateral.

7-2-3 Single-Phase Two-Wire Laterals with Multigrounded Common Neutrals

Figure 7-2 shows a single-phase two-wire lateral with multigrounded common neutral. As shown in the figure, the neutral wire is connected in parallel (i.e., multigrounded) with the ground wire at various places through ground electrodes in order to reduce the current in the neutral wire. I_a is the current in the phase conductor, I_w is the return current in the neutral wire, and I_d is the return current in the Carson's equivalent ground conductor. According to Morrison [1], the return current in the neutral wire is

$$I_n = \zeta_1 I_a \qquad \text{where } \zeta_1 = 0.25 \text{ to } 0.33 \tag{7-28}$$

and it is almost independent of size of the neutral conductor.

In Fig. 7-2, the constant K_R is less than 2.0 and the constant K_X is more or less equal to 2.0 because of conflictingly large D_m (i.e., mutual geometric mean distance or geometric mean radius, GMR) of the Carson's equivalent ground (neutral) conductor.

Therefore, Morrison's data [1] (probably empirical) indicate that

$$VD_{pu,\,1\phi} = \zeta_2 \times VD_{pu,\,3\phi} \qquad \text{where } \zeta_2 = 3.8 \text{ to } 4.2 \tag{7-29}$$

and

$$P_{LS,\,1\phi} = \zeta_3 \times P_{LS,\,3\phi} \quad \text{W} \qquad \text{where } \zeta_3 = 3.5 \text{ to } 3.75 \tag{7-30}$$

Therefore, assuming that the data from Morrison [1] are accurate,

$$K_R < 2.0 \qquad \text{and} \qquad K_X < 2.0$$

the per unit voltage drops and the power losses due to load currents can be approximated as

$$VD_{pu,\,1\phi} \cong 4.0 \times VD_{pu,\,3\phi} \tag{7-31}$$

and

$$P_{LS,\,1\phi} \cong 3.6 \times P_{LS,\,3\phi} \quad \text{W} \tag{7-32}$$

for the illustrative problems.

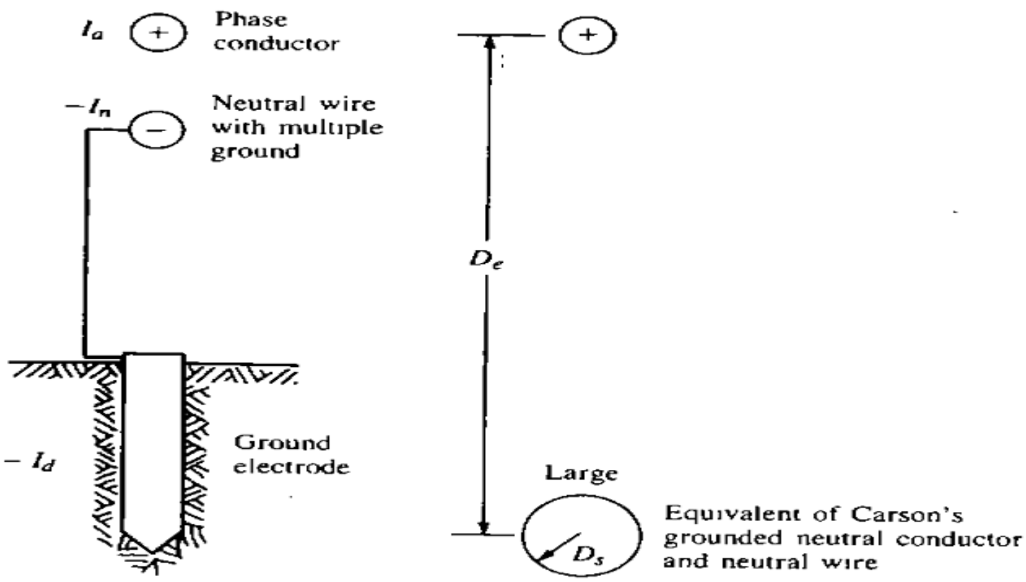

Figure 7-2

7-2-4 Two-Phase Plus Neutral (Open-Wye) Laterals

Figure 7-3 shows an open-wye-connected lateral with two-phase and neutral. Of course, the neutral conductor can be unigrounded or multigrounded, but because of disadvantages the unigrounded neutral is generally not used. If the neutral is unigrounded, all neutral current is in the neutral conductor itself. Theoretically, it can be expressed that

$$\overline{\mathbf{V}} = \overline{\mathbf{Z}}\overline{\mathbf{I}} \tag{7-33}$$

where

$$\overline{V}_a = \overline{Z}_a \overline{I}_a \tag{7-34}$$

$$\overline{V}_b = \overline{Z}_b \overline{I}_b \tag{7-35}$$

It is correct for equal load division between the two phases.

Assuming equal load division among phases, the two-phase plus neutral lateral can be compared to an equivalent three-phase lateral, holding the total kilovoltampere load constant. Therefore

$$S_{2\phi} = S_{3\phi} \tag{7-36}$$

or

$$2V_s I_{2\phi} = 3V_s I_{3\phi} \tag{7-37}$$

from which

$$I_{2\phi} = \tfrac{3}{2} I_{3\phi} \tag{7-38}$$

The voltage-drop analysis can be performed depending upon whether the neutral is unigrounded or multigrounded. If the neutral is unigrounded and the neutral-conductor impedance (Z_n) is zero, the voltage drop in each phase is

$$\mathrm{VD}_{2\phi} = I_{2\phi}(K_R R \cos\theta + K_X X \sin\theta) \quad \mathrm{V} \tag{7-39}$$

$$\text{where } K_R = 1.0$$

$$K_X = 1.0$$

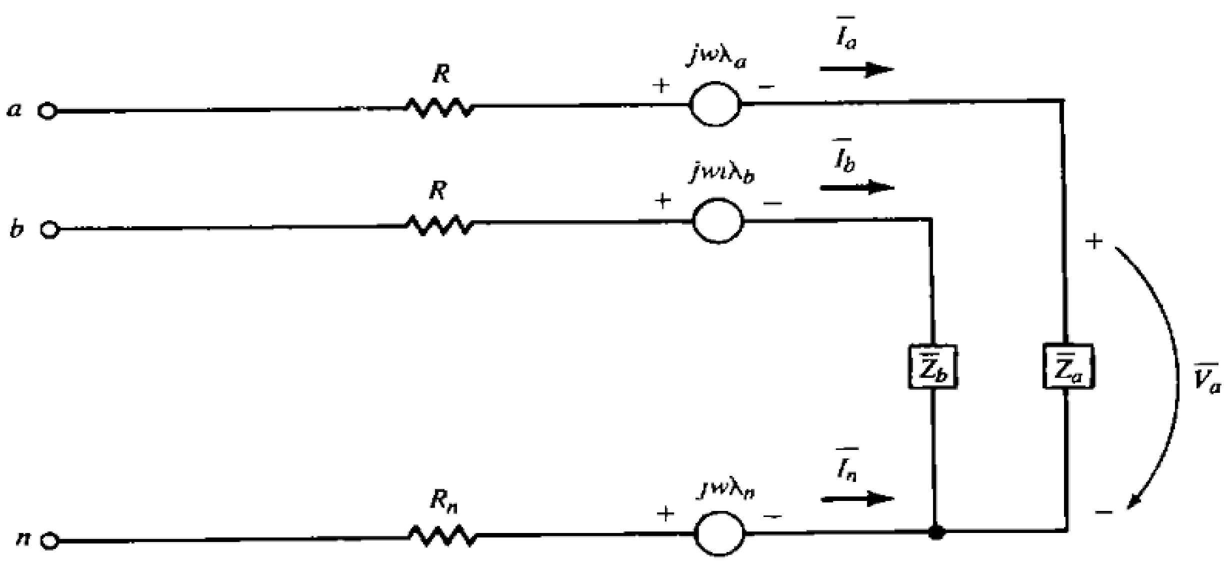

Figure 7-3

Therefore

$$\mathrm{VD}_{2\phi} = I_{2\phi}(R \cos\theta + X \sin\theta) \quad \mathrm{V} \tag{7-40}$$

or substituting Eq. (7-38) into Eq. (7-40),

$$\mathrm{VD}_{2\phi} = \tfrac{3}{2} I_{3\phi}(R \cos\theta + X \sin\theta) \quad \mathrm{V} \tag{7-41}$$

Dividing Eq. (7-41) by Eq. (7-19), side by side,

$$\frac{\mathrm{VD}_{2\phi}}{\mathrm{VD}_{3\phi}} = \frac{3}{2} \tag{7-42}$$

However, if the neutral is unigrounded and the neutral-conductor impedance (Z_n) is larger than zero,

$$\frac{\mathrm{VD}_{2\phi}}{\mathrm{VD}_{3\phi}} > \frac{3}{2} \tag{7-43}$$

therefore in this case some unbalanced voltages are inherent.

However, if the neutral is multigrounded and $Z_n > 0$, the data from Morrison [1] indicate that the per unit voltage drop in each phase is

$$\mathrm{VD}_{\mathrm{pu},\,2\phi} = 2.0 \times \mathrm{VD}_{\mathrm{pu},\,3\phi} \tag{7-44}$$

when a full-capacity neutral is used and

$$\mathrm{VD}_{\mathrm{pu},\,2\phi} = 2.1 \times \mathrm{VD}_{\mathrm{pu},\,3\phi} \tag{7-45}$$

when a reduced-capacity neutral (i.e., when the neutral conductor employed is one or two sizes smaller than the phase conductors) is used.

The power loss analysis also depends upon whether the neutral is unigrounded or multigrounded. If the neutral is unigrounded, the power loss is

$$P_{\mathrm{LS},\,2\phi} = I_{2\phi}^2 (K_R R) \tag{7-46}$$

where $K_R = 3.0$ when a full-capacity neutral is used
$K_R > 3.0$ when a reduced-capacity neutral is used

Based on the data from Morrison [1], the approximate value of this ratio is

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} \cong 1.64 \tag{7-50}$$

which means that the power loss due to load currents in the conductors of the two-phase three-wire lateral with multigrounded neutral is approximately 1.64 times larger than the one in the equivalent three-phase lateral.

Therefore, if $K_R = 3.0$,

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} = \frac{3I_{2\phi}^2 R}{3I_{3\phi}^2 R} \tag{7-47}$$

or

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} = 2.25 \tag{7-48}$$

On the other hand, if the neutral is multigrounded,

$$\frac{P_{\mathrm{LS},\,2\phi}}{P_{\mathrm{LS},\,3\phi}} < 2.25 \tag{7-49}$$

7-3 FOUR-WIRE MULTIGROUNDED COMMON-NEUTRAL DISTRIBUTION SYSTEM

Figure 7-4 shows a typical four-wire multigrounded common-neutral distribution system. Because of the economic and operating advantages, this system is used extensively. The assorted secondaries can be, for example, either (1) 120/240-V

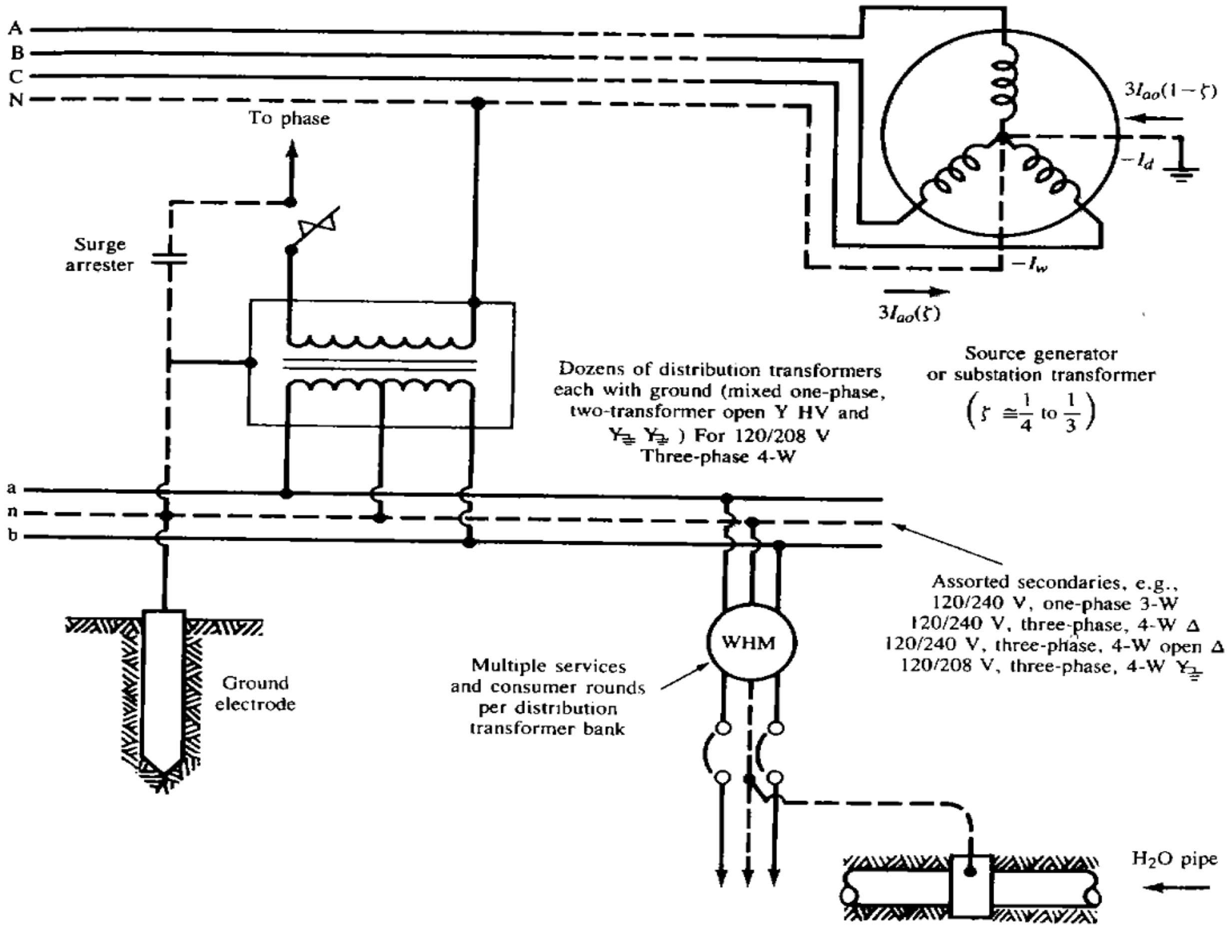

Figure 7-4 A four-wire multigrounded common-neutral distribution system.

single-phase three-wire, (2) 120/240-V three-phase four-wire connected in delta, (3) 120/240-V three-phase four-wire connected in open-delta, or (4) 120/208-V three-phase four-wire connected in grounded-wye. Where primary and secondary systems are both existent, the same conductor is used as the "common" neutral for both systems. The neutral is grounded at each distribution transformer, at various places where no transformers are connected, and to water pipes or driven ground electrodes at each user's service entrance. The secondary neutral is also grounded at the distribution transformer and the service drops. Typical values of the resistances of the ground electrodes are 5, 10, or 15 Ω. Under no circumstances should they be larger than 25 Ω. Usually, a typical metal water pipe system has a resistance value of less than 3 Ω. A part of the unbalanced, or zero sequence, load current flows in the neutral wire, and the remaining part flows in the ground and/or the water system. Usually the same conductor size is used for both phase and neutral conductors.